Some points on graphs:
finitary Poisson processes,
first-shot events, & interventions

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Modeling communication systems as structured point processes

System consists of N entities.

Events occur between entities (N_1, N_2)

Each event occurs at a time point (t)

Directed Graphs

who talks to whom?

Point Processes

Why, when and how do they talk to each other?

Example systems modeled as structured point processes

human communication
Perry & Wolfe (2013); Guo et al., (2015); Zhao et al. (2015)

financial markets
Bowsher (2007); Linderman & Adams (2014); Bacry et al. (2015)

computing networks
Simma et al., (2008); Parikh et al., (2012); Vattani et al., (2015)

neuron spiking activity
Pillow et al., (2008); Dethier et al., (2013); Latimer et al., (2014)

⋮

Background

Poisson Processes (PPs)

$$\mathbb{E}_{S_0 \in S}(\#e) \sim \text{Pois}(\Lambda_{S_0})$$

$$\Lambda_{S_0} = \int_{S_0} \lambda(s)ds$$

$$\lambda(\cdot) \geq 0, \ \exists s \in S \ \text{s.t.} \ \lambda(s) > 0$$

Marked Temporal PPs

$$S = [0, \infty)$$

$$\lambda(t)$$

$$\forall e_i, m(e_i) \rightarrow N_j \in \{N\}$$

Independent PPs

HPPs

$$\lambda(t) = r \qquad \Lambda_{[0,\infty)} = \infty$$

NHPPs

$$\exists\, t_i, t_j \ \text{s.t.} \ \lambda(t_i) \neq \lambda(t_j)$$

Superposition

$$\mathrm{PP}_1\big(\lambda_1(t)\big), \mathrm{PP}_2\big(\lambda_2(t)\big), \mathrm{PP}_1 \perp\!\!\!\perp \mathrm{PP}_2$$

$$\bigcup_{i=1}^{2} \mathrm{PP}_i = \mathrm{PP}\Big(\sum_{i=1}^{2} \lambda_i(t)\Big)$$

Self-exciting Poisson Processes (Hawkes Processes)

Hawkes (1971a,b); Moller & Rasmussen (2005)

$$\lambda(\cdot) \triangleq \lambda(t|e(t_i), m(e(t_i)) \forall t_i < t)$$

Conditional Superposition

Event-induced rate kernels

Hawkes processes on directed graphs

Brillinger (1996); Dahlhaus et. al., (1997); Simma et al. (2010)

Edges encode structural dependencies

Edge-kernels encode semantics

What is missing?

Current models need non-0 base-rates to start cascades

No forward sampling for points on directed graphs

No semantics for disabling nodes on 1st-shot events

No semantics for point interventions on nodes

Current work

Finitary Poisson Processes (FPPs)

$$\int_0^\infty \lambda(s)ds = K, K < \infty$$

$$\mathbb{E}_{[0,\infty)}(\#e) = K$$

$$k \sim \text{Pois}(K) \therefore P(k = 0) > 0$$

Sampling FPPs:

How many events?

$$\int_0^\infty \lambda(s)ds = K, K < \infty$$

$$k \sim \mathrm{Pois}(K)$$

When?

$$e_i \sim \frac{\lambda(t)}{K}$$

Sampling FPPs on graphs

Go to first event $e(t_1)$ on N_i

Induce FPP from edge kernels to N_i's children

Generate events for each child node's new FPP

Union those events with the node's events

Go to next event on entire graph's event set, & repeat

⋮

First-shot events

Time until first arrival: no events before t_1

$$P(t_1|k \geq 1) = (1 - \int_{0,t_1} \frac{\lambda(s)}{K} ds)^k$$

Sometimes 1st arrivals are all that matters…

e.g., acquired immunity, "buy now" bids, death

Point Interventions (☞.)

☞. acts as an independent point event

☞. allows 0 base-rates

0 base-rates allow pure FPP generative graphical models
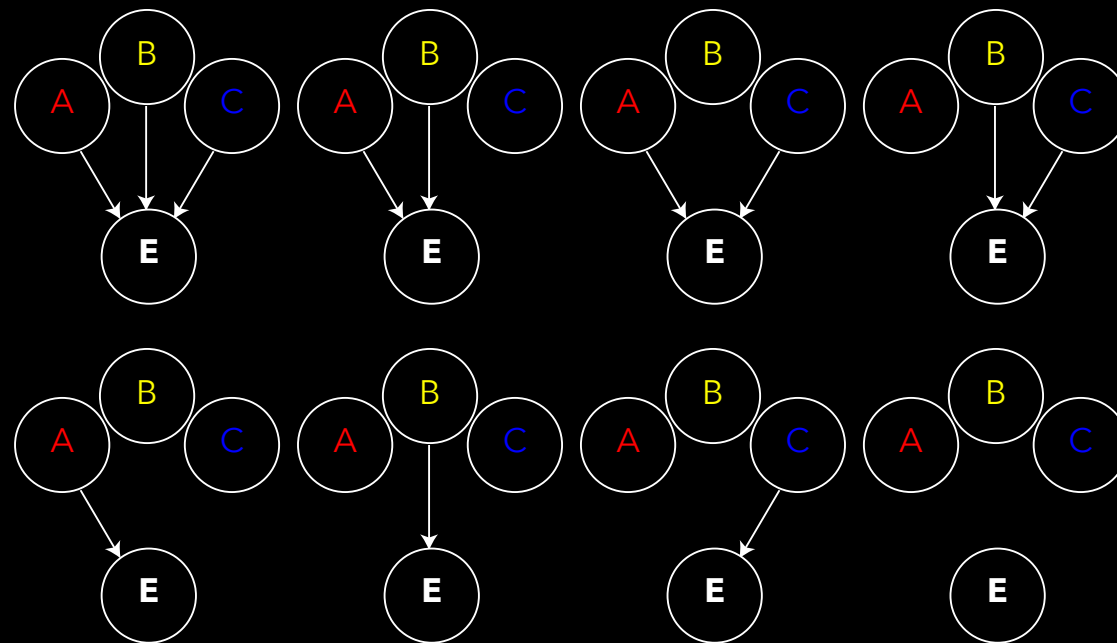
An application

modeling human causal induction
as structure inference

Lagnado & Speekenbrink (2010)
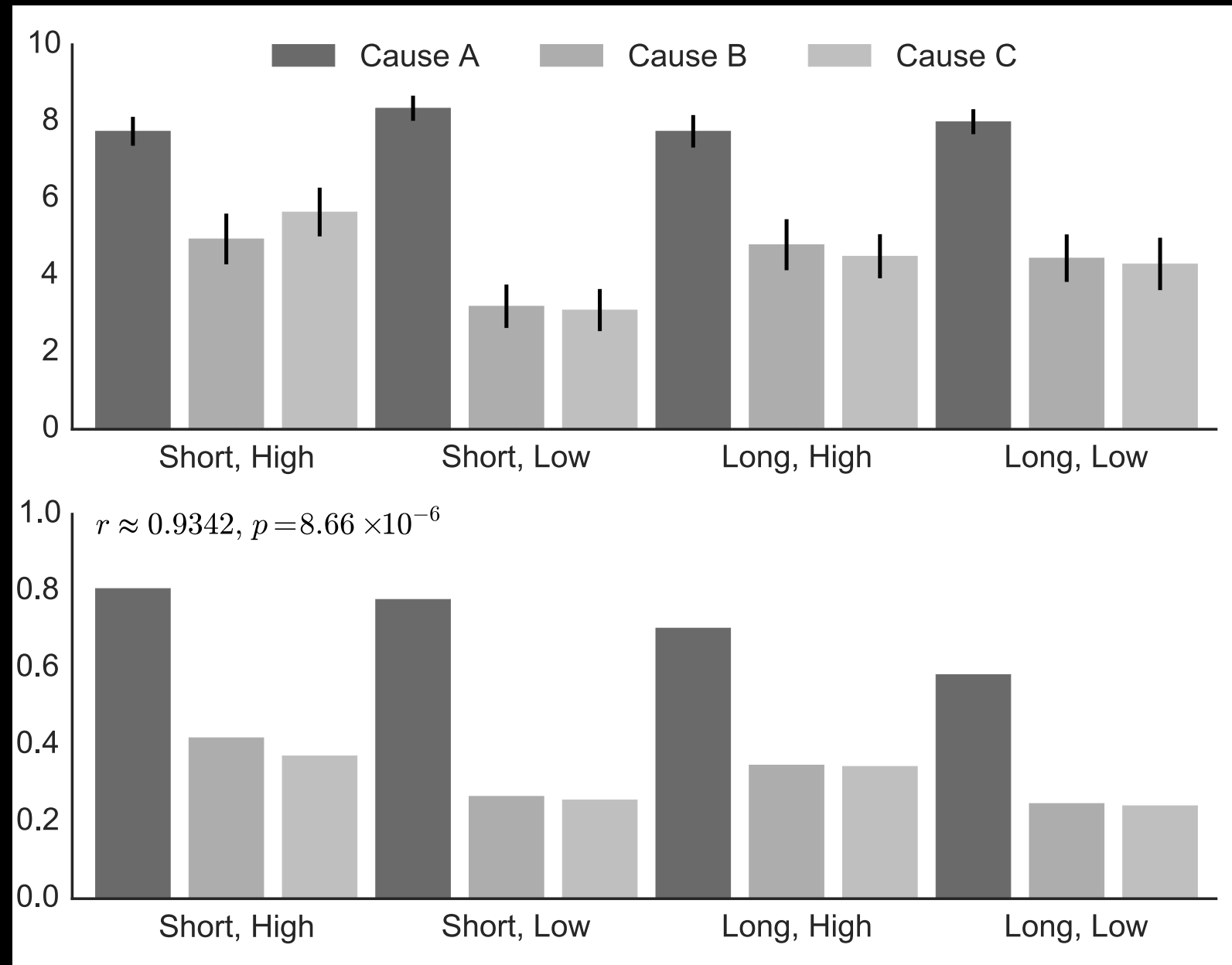
Inferring Causal Structure

4 nodes (A,B,C,**E**) with associated point events

(A,B,C) are independent (☞•) potential causes of **E**

Structure induction as posterior inference over graphs

Pacer & Griffiths (2015)

Posterior inference over graphs predicts human causal judgments

Human Judgments

Bayes with FPPs

Pacer & Griffiths (2015)

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In conclusion

Some Key Points

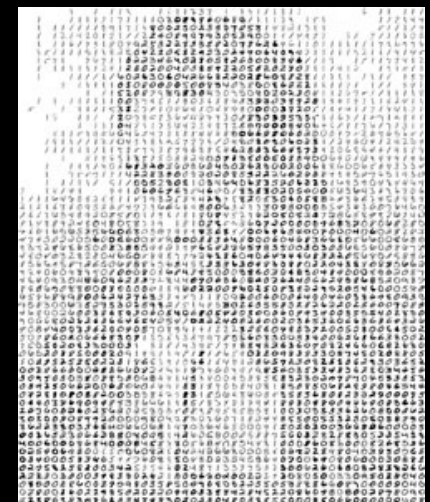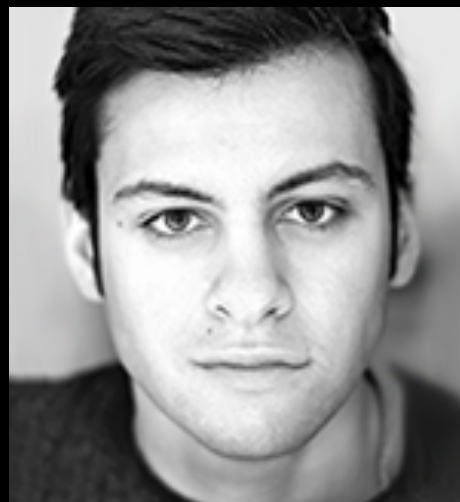graphs + point processes have diverse applications

FPPs allow exact forward sampling for inference

1st-shot events allow modeling non-stationary shifts

point interventions allow 0-base rate, pure FPPs over graphs

point processes as models of human causal induction

Acknowledgements

1045 Thanks!